DETERMINATION OF OPTIMAL COMBINATION OF RECOVERY RATE (ROM) AND LEVEL (CUT-OFF GRADE) BASED ON THE POLYGON METHOD OF RESERVE ESTIMATION

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Background to the Presentation

- A summary of a case study on "Optimizing a depletable mine design, Rio Blanco lateritic nickel deposit" presented in detail in Rudawsky (1986).

- Made small format and textual changes in data tables, and omitted some data which does not affect results.

Objectives of the Presentation

- ✓ Demonstrate one way of determining the combination of annual ore extraction rate (ROM) and level (ore grade); and

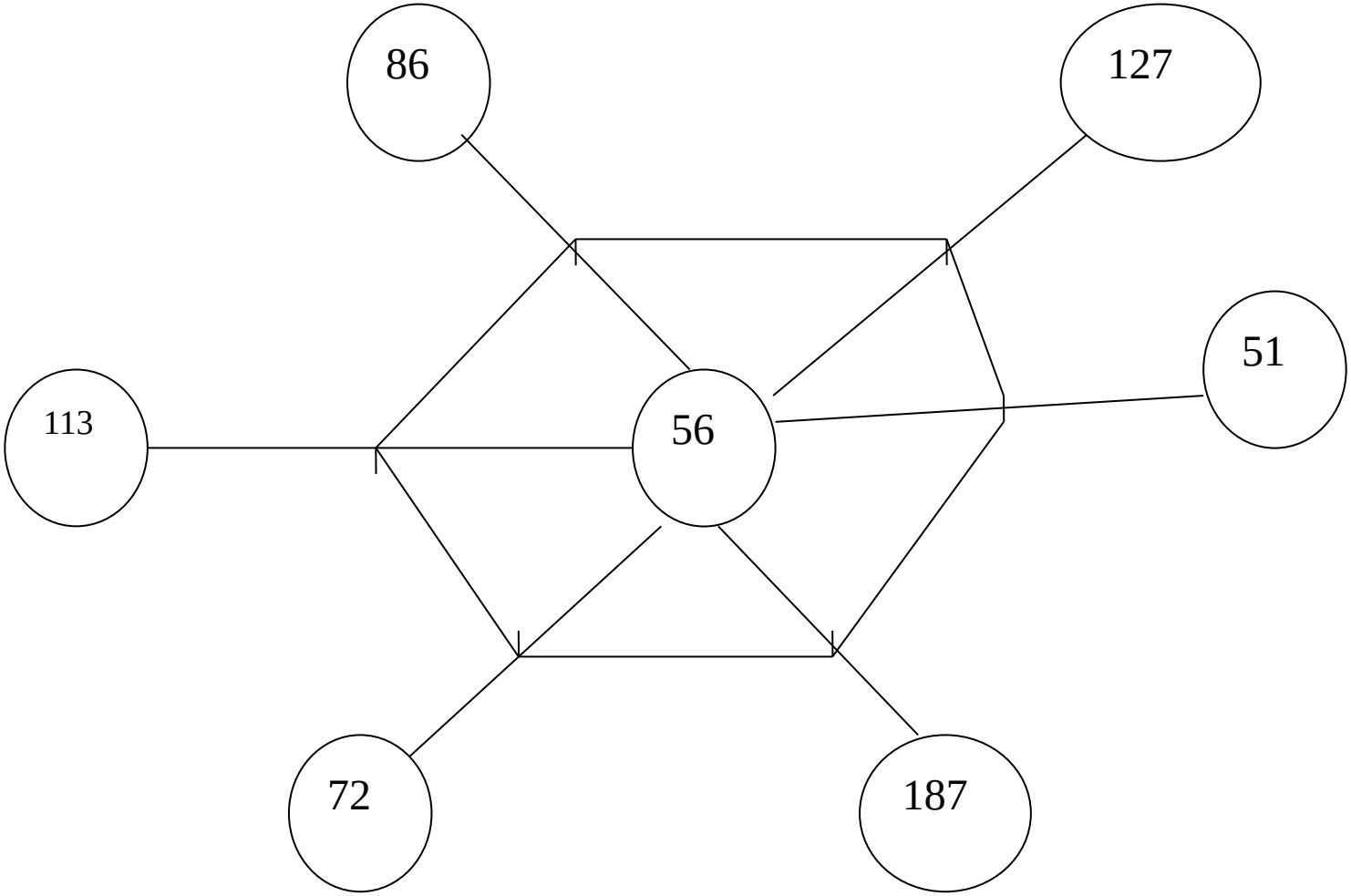
- ✓ Demonstrate how to set negotiation parameters and their quantitative limits especially in a case where, under normal Government policy framework, the project would not be viable.

Presentation Outline

1. Polygon method of reserve estimation
2. Accounting costs at various ROMs and Ore grades
3. Prices and computation of annual and life-time profits at various ROMs and ore grades
4. Cash flows, NPV and IRR
5. Sensitivity analysis, negotiation parameters and limits

Polygon method of reserve estimation

- Exploration - a test pits program numbering 229 pits.
- minimal cut-off grade sought, 0.95%Ni, and 47 pits successful.
- Figure 31 shows how the polygon is formed around pit 56, given adjacent pits 86, 127, 51, 187, 72, 113.


86
127
51
113
56
72
187

Exploratory results for the 47 successful pits

Table 27 (text in red included from outside original table)

Pit No.	Polygon Area (square meters)	Average Thickness (meters)	Volume-Factor (cubic meters)	Metal Assay (%Ni)	Volume-Assay Product
[1]	[2]	[3]	[4] = 2*3	[5]	[6] = 4*5
145	33,005	12.5	412,563	1.26	519,829
121	85,008	9.0	765,072	1.26	963,991
50	94,185	15.0	1,412,775	1.23	1,737,713
51	71,806	1.0	71,806	1.30	93,348
142	91,609	7.0	641,263	1.13	724,627
134	65,527	5.5	360,399	1.01	364,002
53	88,872	15.0	1,333,080	1.10	1,466,388
133	47,625	4.5	214,313	1.18	252,889
164	15,668	6.0	94,008	1.35	126,911
115	58,243	8.5	495,066	1.38	683,190
56	88,872	9.0	799,848	1.22	975,815
55	68,586	19.0	1,303,134	0.95	1,237,977
58	42,021	19.0	798,399	1.48	1,181,631
28	3,542	7.0	24,794	1.41	34,960
144	72,289	16.0	1,156,624	1.35	1,561,442
48	40,894	6.0	245,364	1.21	296,890
60	27,853	14.0	389,942	1.66	647,304
32	16,100	2.0	32,200	1.26	40,572
139	39,767	7.0	278,369	1.26	350,745
27	27,048	11.0	297,528	1.05	312,404
6	17,710	14.0	247,940	1.14	282,652
21	12,397	11.0	136,367	1.49	203,187

Table 27 continued

Pit No.	Polygon Area (square meters)	Average Thickness (meters)	Volume-Factor (cubic meters)	Metal Assay (%Ni)	Volume-Assay Product
[1]	[2]	[3]	[4]	[5]	[6]
61	98,210	14.0	1,374,940	1.37	1,883,668
25	25,277	9.0	227,493	1.31	298,016
24	26,404	18.0	475,272	1.22	579,832
152	60,053	1.0	60,053	1.55	93,082
126	101,108	10.0	1,011,080	1.24	1,253,739
119	83,720	13.0	1,088,360	1.71	1,861,096
136	64,078	27.0	1,730,106	1.20	2,076,127
33	42,665	12.0	511,980	1.30	665,574
34	31,556	7.0	220,892	1.29	284,951
66	76,475	4.0	305,900	1.04	318,136
52	37,996	8.0	303,968	1.33	404,277
104	33,005	5.5	181,528	1.10	199,680
151	25,760	5.0	128,800	1.06	136,528
220	84,847	8.0	678,776	1.06	719,503
188	81,949	4.0	327,796	1.15	376,965
44	84,526	3.0	253,575	1.09	276,397
157	71,806	4.0	287,224	1.25	359,030
210	63,434	4.5	285,453	1.40	399,634
211	95,634	3.0	286,902	1.19	341,413
169	95,956	6.0	575,736	1.31	754,214
214	74,060	2.0	148,120	1.27	188,112
185	79,856	7.0	558,992	1.24	693,150
184	78,246	2.5	195,615	1.14	223,001
170	81,788	2.5	204,470	1.20	245,364
156	47,978	1.0	47,978	1.02	48,938
Total	2,755,013		22,981,863		28,738,894

- Average ore grade (for deposit) = [(Total volume-assay prod) / total volumetric reserves]% = 1.25%

Table 28

Level	**Cut-off Grade (%Ni)**	**No. of polygons**	**Average Ore grade (%Ni)**	**Total Volumetric reserves (cub.m)**	**Conversion factors (cu.m. ⟶ Tons) (density in t/cub.m)**	**Total Reserve tonnages (ore)**
I	0.95	47	1.25	22,981,863	1.675	38,494,620
II	1.05	43	1.28	20,964,452	1.681	35,241,244
III	1.15	34	1.32	17,006,347	1.688	28,706,714

- Assume ore recovery rates in mining and recovery rates in concentration are as given in Table 29.

Table 29

Level	Total reserves (t)	Ore recovery rate in mining (%)	Recoverable ore (t)	Recovery rate in concentration (%) (= plant recovery)
I	38,494,620	93	35,799,997	87
II	35,241,244	90	31,717,119	90
III	28,706,714	85	24,400,707	92

- Table 30 shows the various alternative annual mining rates and the expected life-expectancy of the mine under each level

Table 30

Level	**I**	**II**	**III**
Recoverable Reserves	35,799,997	31,717,119	24,400,707
Annual mining rate q (million t)	Life-Time (yrs)	Life-Time (yrs)	Life-Time (yrs)
3.0	11.9	10.6	8.1
3.5	10.2	9.1	7.0
4.0	9.0	7.9	6.1
4.5	8.0	7.0	5.4
5.0	7.2	6.3	4.9
5.5	6.5	5.8	4.4
6.0	6.0	5.3	4.1
6.5	5.5	4.9	3.8
7.0	5.1	4.5	3.5
7.5	4.8	4.2	3.3

- Assume concentrate level (output) of 20%Ni.

- For each level we compute Concentration Ratio - the amount of ore feed (in tons) required to produce one ton of concentrate output - given by:

$$CR = \frac{Average\ metal\ content\ in\ concentrate}{Average\ grade\ of\ ore * \mathrm{Recovery\ rate\ in\ concentration}}$$

$$CR_I = 20\%Ni/(1.25\%Ni * 0.87) = 18.39\ t.ore/t.concentrate$$

$$CR_{II} = 20\%Ni/(1.28\%Ni * 0.90) = 17.36\ t.ore/t.concentrate$$

$$CR_{III} = 20\%Ni/(1.32\%Ni * 0.92) = 16.47\ t.ore/t.concentrate$$

Accounting Costs

- Total operating costs (TOC) = Total Costs of Mining (TCM) + Concentration Costs (CC)
- TCM vary with q and level, while CC tend to be the same for all levels (we shall assume it anyway), and is assumed to be US$55/ton of concentrate.
- TCM at each level is assumed to be a cubic function of q (a normal assumption for cost functions in economics) as shown in equations below in which TCM is in US$'000 and q is in million tons:

$$TCM_I = 5{,}234 + 1{,}710q - 88q^2 + 15q^3$$

$$TCM_{II} = 6{,}030 + 1{,}507q - 65q^2 + 18q^3$$

$$TCM_{III} = 6{,}980 + 1{,}393q - 58q^2 + 22q^3$$

- The following three panels of Table 31 (Rudawsky, 1986, pp.139-140) (corresponding to the three levels) show:

✓ total mining cost (subst. q into cost equation),

✓ average cost of mining,

✓ feed cost per ton of concentrate,

✓ average total cost per ton of concentrate for the various annual mining rates (ROMs).

- Feed (mining) cost/t.concent. (ACM2) = Concentration ratio (CR) * Cost of mining a t.of ore (ACM1)

- Average total cost of producing a ton of concentrate (ATC) = Feed cost/t.concentrate + CC

Table 31

Level I

Annual Rate	Total cost of mining	Aver.Cost of Mining	Feed cost	Conc. Cost	Average Total Cost
q	TCM	ACM1	ACM2	CC	ATC
[Mill.tons]	$'000	[$/ton ore]	[$/ton conc.]	[$/ton conc.]	[$/ton conc.]
3.0	9,977	3.33	61.24	55.00	116.24
3.5	10,784	3.11	57.16	55.00	112.14
4.0	11,626	2.91	53.45	55.00	108.45
4.5	12,514	2.78	51.14	55.00	106.14
5.0	13,459	2.69	49.50	55.00	104.50
5.5	14,473	2.63	48.39	55.00	103.39
6.0	15,566	2.59	47.71	55.00	102.71
6.5	16,750	2.58	47.39	55.00	102.39
7.0	18,037	2.58	47.39	55.00	102.39
7.5	19,437	2.59	47.66	55.00	102.66

Table 31 cont.

Level II

Annual Rate q [Mill.tons]	Total cost of mining TCM $'000	Aver.Cost of Mining ACM1 [$/ton ore]	Feed cost ACM2 [$/ton conc.]	Conc. Cost CC [$/ton conc.]	Average Total Cost ATC [$/ton conc.]
3.0	10,452	3.48	60.48	55.00	115.48
3.5	11,280	3.22	55.95	55.00	110.95
4.0	12,170	3.04	52.82	55.00	107.82
4.5	13,136	2.92	50.67	55.00	105.67
5.0	14,190	2.84	49.27	55.00	104.27
5.5	15,347	2.79	48.44	55.00	103.44
6.0	16,620	2.77	48.08	55.00	103.08
6.5	18,023	2.77	48.13	55.00	103.13
7.0	19,568	2.80	48.53	55.00	103.53
7.5	21,270	2.84	49.23	55.00	104.23

Table 31 cont.

Level III

Annual Rate q [Mill.tons]	Total cost of mining TCM $'000	Aver.Cost of Mining ACM1 [$/ton ore]	Feed cost ACM2 [$/ton conc.]	Conc. Cost CC [$/ton conc.]	Average Total Cost ATC [$/ton conc.]
3.0	11,231	3.74	61.66	55.00	116.66
3.5	12,088	3.45	56.88	55.00	111.88
4.0	13,032	3.26	53.66	55.00	108.66
4.5	14,079	3.13	51.53	55.00	106.53
5.0	15,245	3.05	50.22	55.00	105.22
5.5	16,547	3.01	49.55	55.00	104.55
6.0	18,002	3.00	49.42	55.00	104.42
6.5	19,626	3.02	49.73	55.00	104.73
7.0	21,435	3.06	50.43	55.00	105.43
7.5	23,446	3.13	51.49	55.00	106.49

- (At this stage it is tempting to make the choice of optimal combination on the basis of least average total cost of producing the concentrate (either q = 6.5 at level I or q = 7.0 at level I).

- (However this only looks at the cost side only, ignoring revenue)

Prices and profits

- Table 32 (Rudawsky, 1986, p.140): 20% Nickel Concentrates Prices, f.o.b.

Annual output (in m. tons of concentrates)	Price (f.o.b) ($/ton of concentrates)
Up to 200,000	140.00
200,000 – 249,999	137.75
250,000 – 299,999	135.40
300,000 – 349,999	132.85
350,000 – 399,999	130.05
400,000 – 449,999	126.70
450,000 – 499,999	123.20

- The following table then gives the for the various mining rates (ROMs) at the three levels of recovery:

✓ output levels,
✓ prices,
✓ revenues,
✓ costs and
✓ profits.

Table 33 : Revenues, Costs, and Profits, Rio Blanco Deposit

Annual Output (million m. tons of ore) (1)	Annual Output (m. tons of concentrates) (2)	f.o.b. Price ($/ton of concentrates) (3)	Total Revenue TR ($) (4)	Marginal Revenue ($/ton) (5)	ATC ($/ton) (6)	TC ($) (7)	Profit per year ($) (8)	Life-Time (yrs) (9)	Profit over the Life-Time ($) (10)
			Level I						
3.0	163,132	140.00	22,838,480	140.00	116.24	18,962,463	3,876,016	11.9	46,124,590
3.5	190,321	140.00	26,644,940	122.00	112.14	21,342,596	5,302,343	10.2	54,083,898
4.0	217,510	137.75	29,962,002	137.75	108.45	23,588,959	6,373,043	9.0	57,357,382
4.5	244,698	137.75	33,707,149	114.25	106.14	25,972,245	7,734,903	8.0	61,879,226
5.0	271,887	135.40	36,813,499	135.40	104.50	28,412,191	8,401,308	7.2	60,489,414
5.5	299,076	135.40	40,494,890	104.80	103.39	30,921,467	9,573,422	6.5	62,227,245
6.0	326,264	132.85	43,344,172	96.45	102.71	33,510,575	9,833,597	6.0	59,001,579
6.5	353,453	130.05	45,966,562	130.05	102.39	36,190,052	9,776,509	5.5	53,770,801
7.0	380,642	130.05	49,502,492	79.80	102.39	38,973,934	10,528,557	5.1	53,695,643
7.5	407,830	126.70	51,672,061		102.66	41,867,827	9,804,233	4.8	47,060,319
			Level II						
3.0	172,811	140.00	24,193,540	124.25	115.45	19,951,029	4,242,510	10.6	44,970,606
3.5	201,613	137.75	27,772,190	137.75	110.95	22,368,962	5,403,228	9.1	49,169,371
4.0	230,414	137.75	31,739,528	116.60	107.82	24,843,237	6,896,291	7.9	54,480,695
4.5	259,217	135.40	35,097,981	135.40	105.67	27,391,460	7,706,521	7.0	53,945,644
5.0	288,018	135.40	38,997,637	107.35	104.27	30,031,636	8,966,000	6.3	56,485,800
5.5	316,820	132.85	42,089,537	132.85	103.44	32,771,860	9,317,676	5.8	54,042,521
6.0	345,622	132.85	45,915,882	96.45	103.08	35,626,715	10,319,166	5.3	54,691,579
6.5	374,424	130.05	48,693,841	83.15	103.13	38,614,347	10,079,493	4.9	49,389,520
7.0	403,226	126.70	51,088,734	126.70	103.53	41,745,987	9,342,746	4.5	42,042,357
7.5	432,028	126.70	54,737,947		104.23	45,030,278	9,707,669	4.2	40,772,207
			Level III						
3.0	182,149	140.00	25,500,860	124.25	116.66	21,249,502	4,251,358	8.1	34,435,997
3.5	212,508	137.75	29,272,977	137.75	111.88	23,775,395	5,497,582	7.0	38,483,073
4.0	242,866	137.75	33,454,791	116.60	108.66	26,389,819	7,064,972	6.1	43,096,329
4.5	273,224	135.40	36,994,529	109.90	106.53	29,106,552	7,887,976	5.4	42,595,071
5.0	303,582	132.85	40,330,868	132.85	105.22	31,942,898	8,387,970	4.9	41,101,052
5.5	333,940	132.85	44,363,929	99.25	104.55	34,913,427	9,450,502	4.4	41,582,208
6.0	364,299	130.05	47,377,084	130.05	104.42	38,040,101	9,336,982	4.1	38,281,627
6.5	394,657	130.05	51,325,142	83.15	104.73	41,332,427	9,992,714	3.8	37,972,314
7.0	425,015	126.70	53,849,400	74.20	105.43	44,809,331	9,040,069	3.5	31,640,239
7.5	455,373	123.20	56,101,953		106.49	48,492,670	7,609,282	3.3	25,110,631

(2) = (1) ÷ CR; (4) = (2) x (3); (5) = $\frac{TR}{(2)}$; (7) = (6) x (2); (8) = (4) - (7); (10) = (8) x (9)

- (At this stage it is also tempting to choose the q and level with the highest yearly profits (q=7.0, level I) or the highest life-time profits (q=5.5, level I)).

- However, we need to take into account time-value of money, depreciation, depletion allowances, tax

- **Important observation**: these results show that:

✓ annual profits are maximized at higher rates of recovery (q)

✓ while life-time profits are maximized at a lower rate of extraction

Cash Flows and Present Value Computations

- **Capital investment**

✓ Replaceable equipment , replaced at end of 5 years. Replacement is at historic cost.

✓ life-time long investment

- Depreciation allowance – straight line method
- Percentage depletion allowance at 15%
- Royalty of 12.5% of gross sales income
- 50% corporate tax rate applied on accounting profits
- There is some salvage value
- Minimum rate of return (discount rate) of 20%

- Table 34 shows capital investments and salvage values for the various extraction rates (ROMs):

Annual Rate q (million t of ore)	Expected life-time (yrs)	Capital investment requirements (US$'000)	% salvage	Salvage value
3.0.	Life-time 5 yrs	3,750 9,983 13,733	20% 20%	750 1,997 2,747
3.5	Life-time 5 yrs	3,850 10,783 14,633	23% 20%	886 2,157 3,043
4.0	Life-time 5 yrs	4,394 12,992 17,386	25% 20%	1,099 2,598 3,697
4.5	Life-time 5 yrs	4,515 13,481 17,996	28% 20%	1,264 2,696 3,960
5.0	Life-time 5 yrs	4,532 14,074 18,606	31% 20%	1,405 2,815 4,220
5.5	Life-time 5 yrs	4,893 16,247 21,140	34% 20%	1,664 3,249 4,913
6.0	Life-time 5 yrs	5,129 17,413 22,542	39% 20%	2,000 3,483 5,483
6.5	Life-time 5 yrs	5,530 17,756 23,286	44% 20%	2,433 3,511 5,944
7.0	Life-time 5 yrs	24,037	27%	6,490
7.5	Life-time 5 yrs	25,216	30%	7,565

- Depreciation allowances are summarized in Table 35

Table 35 summarizes the depreciation allowances for the varying rates and levels of extraction.

Table 35 : Annual Depreciation allowances, Rio Blanco Deposit

Annual Rate q (million m. tons of ore)	Depreciable Investment (thousands of U.S. dollars)	Level I Life-time (in years)	Level I Annual Deprec. (thousands of U.S. $)	Level II Life-time (in years)	Level II Annual Deprec. (thousands of U.S. $)	Level III Life-time (in years)	Level III Annual Deprec (thousands U.S. $
3.0	3,000 7,986	11.9 5.0	252 1,597 1,849	10.6 5.0	283 1,597 1,880	8.1 5.0	370 1,597 1,967
3.5	2,964 8,626	10.2 5.0	291 1,725 2,016	9.1 5.0	326 1,725 2,051	7.0 5.0	423 1,725 2,148
4.0	3,295 10,394	9.0 5.0	366 2,079 2,445	7.9 5.0	417 2,079 2,496	6.1 5.0	540 2,079 2,619
4.5	3,251 10,785	8.0 5.0	406 2,157 2,563	7.0 5.0	464 2,157 2,621	5.4 5.0	602 2,157 2,759
5.0	3,127 11,259 14,386	7.2 5.0	434 2,252 2,686	6.3 5.0	496 2,252 2,748	4.9 {	2,936
5.5	3,229 12,998 16,227	6.5 5.0	497 2,600 3,097	5.8 5.0	557 2,600 3,157	4.4 {	3,688
6.0	3,129 13,930 17,059	6.0 5.0	522 2,786 3,308	5.3 5.0	590 2,786 3,376	4.1 {	4,161
6.5	3,097 14,245 17,342	5.5 5.0	563 2,849 3,412	4.9 {	3,539 {	3.8	4,564
7.0	17,547	5.0	3,509	4.5	3,899	3.5	5,013
7.5	17,651	4.8	3,677	4.2	4,203	3.3	5,349

- At this stage all data necessary for cash flow development are available.

- For level I at the mining rate (ROM) of 3 million tons Table 36 gives the cash flow, the NPV and the IRR.

- There are 30 different such cash flow tables since there are three levels, each level with 10 alternative ROMs.

Table 36 :Annual Cash Flows, N.P.V. @ 20%, and D.C.F. Rate of Return, Rio Blanco Deposit

Rate: 3.0 million m. tons of ore/year, Level I; Life-time: 11.9 years.
(in thousands of U.S. Dollars)

Item/Year	0	(per year) 1-4	5	(per year) 6-9	10	11	12
(1) TR	-	22,838	22,838	22,838	22,838	22,838	20,55
(2) Royalty	-	2,855	2,855	2,855	2,855	2,855	2,57
(3) Working Inter.	-	19,983	19,983	19,983	19,983	19,983	17,98
(4) TC (excl. depr.)	-	17,113	17,113	17,113	17,113	17,113	15,40
(5) Depre. Allowance	-	1,849	1,849	1,849	1,849	1,849	1,66
(6) Balance I	-	1,021	1,021	1,021	1,021	1,021	91
(7) Deplet. Allowance	-	511	511	511	511	511	45
(8) Taxable Income	-	510	510	510	510	510	45
(9) Tax Liability	-	255	255	255	255	255	22
(10) After Tax Prof.	-	255	255	255	255	255	23
(11) Noncash Add.	-	2,360	2,360	2,360	2,360	2,360	2,12
(12) Cap. Investment	13,733	0	9,983	0	9,983	0	
(13) Salvage Value	-	0	1,997	0	1,997	0	2,74
(14) Annual Cash Flow*	(13,733)	2,615	(5,371)	2,615	(5,371)	2,615	5,10
(15) Dis. Factor @ 20%	1.000	2.589	0.402	1.041	0.162	0.135	0.11
(16) N.P.V. @ 20%	(13,733)	6,770	(2,159)	2,721	(868)	352	57
(17) Cum. N.P.V. @ 20%	(13,733)	(6,963)	(9,122)	(6,401)	(7,269)	(6,917)	(6,34
(18) D.C.F. R.O.R.			4.54%				

(1) = data from Table 33; (2) = (1) ÷ 8; (3) = (1) - (2); (4) = data from Table 3
(5) = data from Table 35; (6) = (3) - (4) - (5); (7) = the smaller value of (3) x .15; or (6) ÷ 2; (8) = (6) - (7); (9) = (8) x .50; (10) = (8) - (9); (11) = (5) + (7); (12) = data from Table 34; (13) = data from Table 34; (14) = (10) + (11) - (1 + (13). Items (1), (4), and (5) for the last year of operations are multiplied by the fraction 0.9.

*A number in parentheses indicates a negative value.

- Table 45 gives a summary of the NPVs and the IRR for all the levels and all the ROMs:

The results of all thirty alternative combinations of rates and levels are summarized in Table 45 and Figure 32 below:

Table 45 :Summary Results, N.P.V. @ 20%, and D.C.F. Rate of Return, Rio Blanco Deposit

Annual Rate of q (million m. tons of ore/year)	Level I N.P.V. (thousand U.S. Dollars)	Level I R.O.R. (% per year)	Level II N.P.V.	Level II R.O.R.	Level III N.P.V.	Level III R.O.R.
3.0	(6,345)	4.54	(5,770)	4.89	(5,796)	4.44
3.5	(4,024)	9.87	(3,442)	12.37	(4,533)	7.25
4.0	(3,042)	14.31	(2,808)	14.19	(4,614)	7.66
4.5	(1,432)	17.20	(2,802)	13.81	(4,664)	6.54
5.0	(1,934)	16.03	(2,447)	14.34	(767)	18.24
5.5	(2,535)	14.98	(4,514)	9.75	(969)	17.93
6.0	(4,287)	11.22	(5,329)	7.64	(2,661)	14.45
6.5	(7,021)	4.62	(1,616)	17.05	(2,364)	14.95
7.0	(1,197)	17.92	(4,361)	11.94	(5,092)	9.04
7.5	(4,211)	12.90	(5,177)	10.60	(8,051)	3.13

- All the thirty combinations of q and levels are not viable at the required minimum return of 20%.

- Now, what is the way forward – are we going to give up on the project just like that?

Sensitivity analysis

- Management realized that if they were to have royalty rate reduced from 12.5% to 10% several combinations would yield positive NPVs and IRRs>20% (do the exercise).

- (Other parameters that may be changed in sensitivity analysis include:
  - ✓ CIT
  - ✓ Depreciation method (from straight-line to accelerated)
  - ✓ Incentives, such as tax holidays, etc.)

- These then become subjects for negotiation with government so that the project may become viable.

Reference

1. Rudawsky, O. (1986). Mineral Economics – Development and Management of Naturals Resources. *Developments in Economic Geology, 20*. Elsevier. Amsterdam.

THANK YOU!!